

# Statistical Assessment of a Limnological Data Set

by

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## ABSTRACT

In a study of Wisconsin Lakes, to examine the effects upon water quality of imposition of a ban on detergent phosphorus, the design protocol employed the concept of test lakes and reference lakes. A pairing was made of each test lake with a reference lake having as many similar characteristics as possible with the test lake except for a loading of phosphorus from municipal wastewater effluent or septic tank seepage. The responses measured for each lake were physical, chemical and biological in nature. Measurements were taken both before and after imposition of the ban. To estimate the potential effect of the ban, three forms of statistical models were used; (i) for each test lake a model using the reference lake variable as a covariate and the ban as a classification variable, (ii) a comprehensive model for all of the lakes combined using the reference lakes as covariates and the test lakes as dummy variables, and (iii) multivariate models providing multiple comparison estimates for pre- and post-ban differences. The advantage to the paired lake approach is the potential for variance reduction, and an examination of this was made for several data sets. In this paper are discussed the comparisons of the modeling procedures as well as estimates of the "ban effects." Also presented are some of the observed distributional characteristics of the measured responses.

## INTRODUCTION

The growth of algae is, to a large extent, regulated by the presence of the macronutrients nitrogen and phosphorus in the water column (Hutchinson, 1957, Wetzy, 1975). Excess growth can degrade water quality by reducing clarity, adding noxious odors and taste to the water, hampering motorboat movement, and reducing overall aesthetic quality. Of the macronutrients, phosphorus is most frequently "limiting", i.e. the amount of phosphorus input to a water body is the regulating factor in photosynthetic production (Likens, 1972, Schindler, 1977). Phosphorus is an important ingredient in laundry detergents, serving as a "builder" by, among other things, reducing water hardness. As a means of reducing the load of phosphorus to both municipal and private wastewater treatment systems bans prohibiting the presence of phosphorus in laundry detergents have

been imposed in numerous locations around the United States. Although a reduction in treatment plant loadings of phosphorus have been monitored in some of these areas, mixed reviews have appeared as to the effectiveness of detergent phosphorus bans in subsequently improving water quality in these locales (Pieczonka and Hopson, 1974, Bell and Spacie, 1978, Hartig and Horvath, 1982, Runke, 1982, Maki, Porcella and Wendt, 1984).

The state legislature of Wisconsin enacted a multi-year detergent phosphorus ban which became effective on 1 July 1979 and was in effect to 30 June 1982. The Soap and Detergent Association initiated a lake study program in 1978 and continued it through 1983 in order to determine the effectiveness of the ban. The study looked at physical, chemical and biological parameters from the study lakes to determine if any changes in these were resultant from imposition of the ban. An assumption accepted, and borne out throughout the literature, was the strong relationship between phosphorus concentrations and a number of other lake water quality parameters.

Typically, trend analysis of water quality data is hampered by several factors, among them missing values, values below detection limits, seasonality, and the non-normality of the parameter distributions (Hirsch, Slack and Smith, 1982, Van Belle and Hughes, 1984). It has also been reported that an extensive data record is necessary in the assessment of lake restoration programs in order to increase the statistical power level if parametric tests are used (Trautmann, et. al., 1982). As a result, non-parametric statistical methods are usually employed to determine time related variations in water quality. These studies, however, assume that a monitoring record is available only for a limited number of lakes or for only those lakes which are impacted by phosphorus control measures.

Considering that the imposition of a detergent phosphorus ban was an experiment in improving water quality, two groups of lakes were selected for investigation. The experimental group, or "test" lakes, were those lakes within the state of Wisconsin that were determined to be receiving a significant percentage of their phosphorus loading as sewage effluent, from either public or private treatment systems. These lakes would therefore be the most likely to be impacted by a reduction in phosphorus concentration from these sources. The control group, or "reference" lakes, were lakes determined not to be impacted by sewage eff-

luent. By coincidentally monitoring reference lakes a baseline would be established reflecting only natural fluctuations in water quality occurring over time, those chiefly a function of climatic conditions (i.e. temperature, rainfall amounts and frequency). An overall temporal trend in water quality data observed upon the test lakes, which significantly deviated from any observed upon the reference lakes, could then be ascribed as a function of imposition of the detergent phosphorus ban.

### MONITORING METHODOLOGY

In considering lakes to be included in the monitoring program, preference was given to those for which historical information was available from sources such as the National Eutrophication Survey (NES) or the Wisconsin Department of Natural Resources (WDNR) Quarterly Monitoring Program. Consideration in terms of size, depth, and hydraulic residence time followed NES selection criteria (NES, 1974). The locations of the lakes selected for the study are shown in Figure 1. The apparent concentration of study lakes in the northern part of the state is consistent with the actual partitioning of lakes within the state (WDNR, 1975). Groups of lakes fall within regional boundaries set by the WDNR and corresponding to bedrock and glacial geology as well as soil cover (Lillie and Mason, 1983). These groups include test lakes and their corresponding reference lake. In the analysis, test lakes Butternut, Elk and Balsam were paired with reference lake Teal. These lakes are situated in granite soils underlain by a sandstone bedrock (Prescott, 1962). Test lakes Moss, Enterprise and Townline were paired with reference lake Little Bearskin; all are surrounded by sandy or silty soil and underlain by sandstone. Test lake Swan is paired with reference lake Fish; both are located in the alkaline soil of the southern regions of the state and are underlain by limestone.

Limnologic, morphologic and drainage basin characteristics of the lakes are summarized in Table 1. Reference lakes are geographically proximate to their test lakes and it may be noted from Table 1 that, in several cases, morphological dissimilarities are minimal between test-reference lake pairs. Though not "pristine" (residences are located along the lake shore), the reference lakes have the least amount of drainage basin area devoted to shoreline development. The extent of impactation by



Figure 1. Locations of the Wisconsin Study Lakes

sewage effluent upon the test lakes is listed in Table 2. The determination that Balsam, Moss and Enterprise lakes were not impacted by effluent phosphorus was made upon a reevaluation of nutrient loadings conducted after the monitoring study. At the time the study was initiated, in 1978, the phosphorus removal capabilities of municipal land treatment systems and private septic tank tile field systems were in question. These lakes were maintained as test lakes throughout the analysis since they did differ from their respective reference lakes by having effluent land treatment systems within their watersheds and, therefore, could be used to verify the phosphorus removal capabilities of these types of systems. Since the detergent

TABLE 1

Limnological, Morphological, and Drainage Basin Characteristics of the Study Lakes.

Lake	County	Surface Area (ha.) <sub>1</sub>	Volume (10**6 cu m.) <sub>2</sub>	Mean Depth (m) <sub>3</sub>	Max Depth (m)	Mean Hydraulic Residence Time (days) <sub>4</sub>	Number of Tributaries		Immed. Drain. Basin Area (sq. km.) <sub>6</sub>	No. of Residences in 1981 <sub>7</sub>
							In <sub>5</sub>	Out		
Butternut	Price	407	17.10	4.2	10.0	180	4	1	8.5	265
Elk	Price	36	0.55	1.5	8.0	<5	1	1	3.8	7
Balsam	Washburn	119	8.74	7.3	15.0	70	2	1	9.6	24
Teal	Sawyer	425	16.15	3.8	9.0	210	2	1	9.7	137
Moss	Vilas	79	2.36	3.0	9.0	900	0	1	3.0	40
Townline	Oneida	62	2.15	3.5	6.0	220	2	1	1.7	71
Enterprise	Langlade	204	7.26	3.6	8.0	620	1	1	10.9	124
Little Bearskin	Oneida	66	1.57	2.4	8.0	50	1	1	5.0	43
Swan	Columbia	164	16.03	9.8	25.0	160	1	1	21.3	104
Fish	Dane	102	6.34	6.2	19.0	1410	0	0	7.7	66

(1) Source - Wisconsin Department of Natural Resources (1981)

(2) Volumes estimated planimetrically using depth contours from maps prepared by The Clarkson Company, Kauksuna, WI

(3) Lake volume divided by surface area

(4) Lake volume divided by the mean annual flow

(5) Intermittent streams are not listed as tributaries

(6) Source - Wisconsin Department of Natural Resources (1975)

(7) Visual survey conducted by the Environmental Research Group, Inc., St. Paul MN.

(Note: a resort was counted as equivalent to 20 residences, a scout camp equivalent to 40 residences)

phosphorus ban was intended to impact lakes which would be considered candidates for nutrient reduction measures, such as a lake possessing an effluent discharge within its watershed, the effect of the ban upon Balsam, Moss and Enterprise lakes would be relevant to the overall success of the ban.

The Wisconsin lakes were monitored from 1978 through 1982. Only reference lakes were monitored during 1979, the year the ban was initiated. Monitoring of Fish Lake was discontinued in 1981 and, hence, data from Fish Lake is not included in the statistical analyses. Field trips to the lakes occurred between ice-out (late April to mid-May) and fall overturn (late October to early November). The interval between sampling was typically four weeks although samples were collected every two weeks during the summer months (July and August). Samples and measurements were taken, on all of the lakes, at the location of the deepest point and at one or two other locations, depending upon the

TABLE 2. Extent of Wastewater Treatment Within Study Lake Basins.

Lake	Name of Municipal WWTP	Final Application of Treated Wastewater	Phosphorus Load	
			kg/yr	% of Total Load
Elk	Phillips	Direct Discharge to Lake	1660	22
Butternut	Butternut	Indirect Discharge to Surface Water	480	19
Swan	Pardeeville	"	1730	39
Townline	Three Lakes	"	54	8
Balsam	Birchwood	Land Disposal	0	0
Moss	Lac du Flambeau	"	0	0
Enterprise	---	Septic tank/Tile field	0	0

Note: About 30% of wastewater phosphorus may be assumed to come from detergents.

morphology of the lake.

Transparency was measured using a standard Secchi disk. Profile measurements were made at one meter depth increments for temperature, dissolved oxygen, and conductivity. An integrated two meter sample of the epilimnion was obtained using a 37 mm (I.D.) PVC pipe. Aliquots of the integrated sample were stored in amber Nalgene bottles at 4° C and earmarked for specific analyses. Chemical analysis of the samples was typically initiated within 48 hours. Total Phosphorus determinations followed persulfate digestion (Menzyl and Corwin, 1965); the colorimetric reaction involved reduction using ascorbic acid (Murphy and Riley, 1962). Chlorophyll-a was determined using trichromatic methods (APHA, 1976).

#### Temporal Variation of the Data

Temporal plots of the monitoring data, such as that presented in Figure 2, evidence the amount of variability present in water quality records of either physical or chemical parameters. Yearly trends in any of the monitored parameters were difficult to discern from the plots. However, a degree of "tracking", a synchronous correspondance between plots for test and reference lake pairs, could be ascertained in several cases. The obvious imprecision of any subjective determinations made upon the data set, however, lead to the statistical methodology employed.

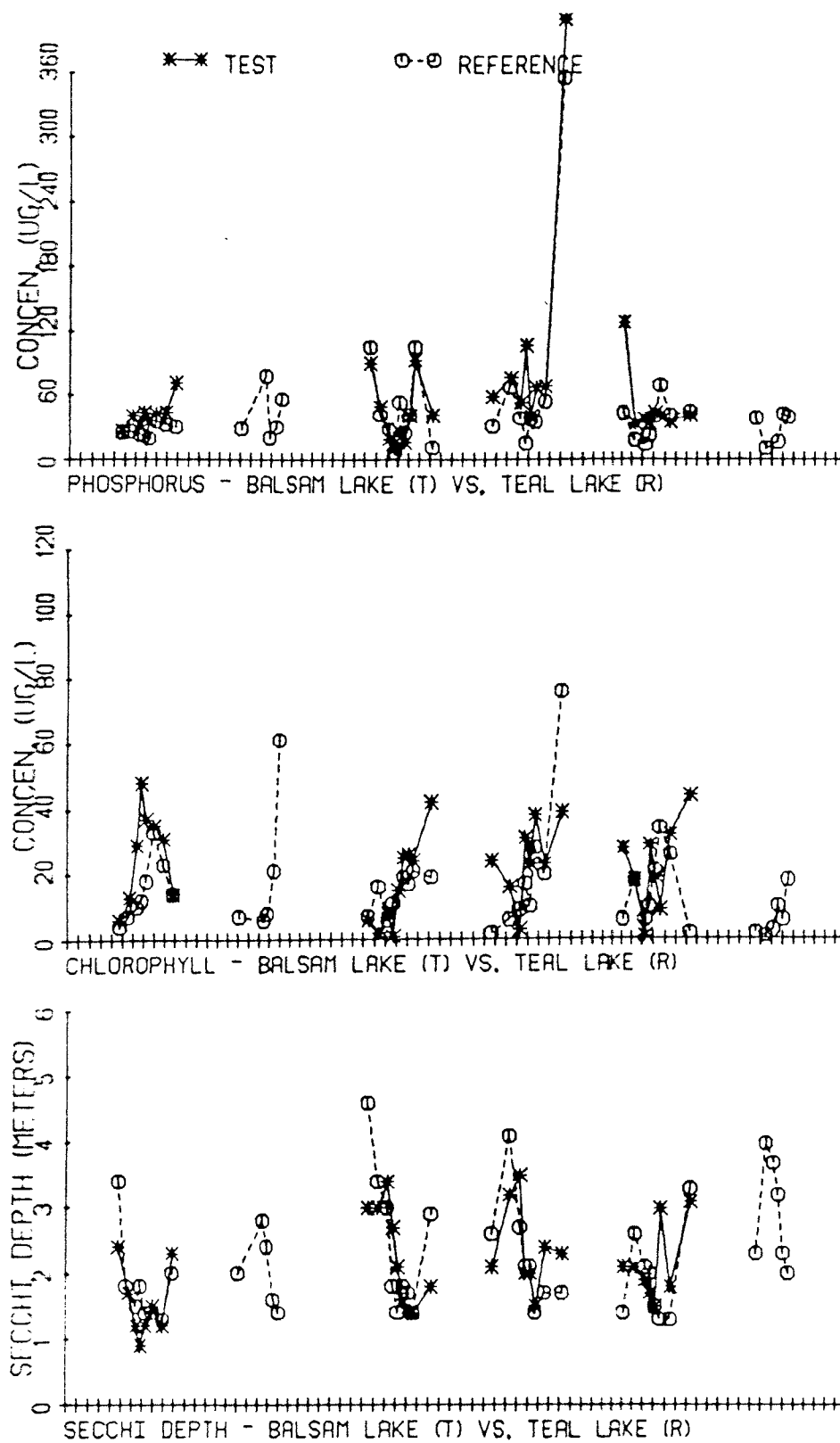


Figure 2. Temporal Variation of the Water Quality Data

## STATISTICAL METHODOLOGY AND RESULTS

For examination of a potential ban effect, three types of statistical analysis were used: (i) covariance analysis for each test lake separately, (ii) combined covariance analyses for all test lakes, and (iii) multivariate analysis obtaining multiple comparison estimates for pre- and post-ban differences of interest. All analyses were performed using logarithmic transformations of the original lake data, a scale of measurement strongly supported by earlier lake data distribution studies.

### Covariance Analysis for Individual Test Lakes

For each test lake, a covariance analysis was performed using a model of the form:

$$\log y_t = \beta_0 + \beta_1 \log y_r + \beta_2 B + \epsilon$$

where  $y_t$  represents a test lake observation,  
 $y_r$  represents a corresponding reference lake observation,  
 $B$  is a 0, 1 indicator variable indicating a pre- or post-ban observation.

One can think of this model permitting variance reduction of the test lake data, due to their association with the reference lake data obtained under similar background conditions, thus allowing a potential difference due to the ban to be detected with improved sensitivity. Table 3 lists the salient features of the covariance analyses for the logarithms of the responses for the individual test lakes.

### Covariance Analysis for All Test Lakes

The estimate for the change in the intercept associated with the post-ban period is the feature of greatest interest. Only for Elk and Townline Lakes for secchi disc depth is this change statistically significant. Part of this may be attributed to the small sample sizes and the large variability, encouraging an examination of the test lakes simultaneously using an "indicator variable" approach. This analysis is described next.

Assuming that the test lake-reference lake relationship is similar for all the lakes, improved sensitivity for ban-effect detection is provided by a model that simultaneously considers



TABLE 3. Covariance Analysis for Individual Test Lakes.

Reference Lake	-----Fish-----									-----Teal-----								
	Swan			Balsam			Butternut			Elk								
	TP	SD	CHLA	TP	SD	CHLA	TP	SD	CHLA	TP	SD	CHLA	TP	SD	CHLA	TP	SD	CHLA
Intercept $\beta_0$	1.89	1.52	1.11	.89	.27	1.04	1.29	-.06	.43	1.36	.14	.33						
Post-Ban Change $\beta_2$	.08	.17	.31	.00	.12	-.20	.00	.13	-.01	-.05	.17	.28						
Standard Error of Change	.13	.10	.20	.09	.05	.19	.08	.06	.21	.08	.05	.15						
	*			*			*			*			*			*		
Slope $\beta_1$	-.13	-.80	-.15	.51	.55	.28	.38	.68	.80	.30	.26	.48						
Standard Error	.28	.35	.33	.15	.13	.27	.13	.18	.33	.14	.13	.20						
R-squared	.00	.08	.01	.31	.42	.04	.25	.39	.18	.13	.20	.14						
$\Delta$ R-squared	.02	.11	.10	.00	.12	.04	.00	.09	.00	.00	.27	.11						

Reference Lake	-----Little Bearskin-----								
	Enterprise			Moss			Townline		
	TP	SD	CHLA	TP	SD	CHLA	TP	SD	CHLA
Intercept $\beta_0$	-.41	-.01	.13	-.28	.37	.08	.85	.31	.97
Post-Ban Change $\beta_2$	.18	.00	.16	.02	.06	.07	.00	.22	-.10
Standard Error of Change	.12	.07	.13	.10	.05	.09	.08	.05	.12
	*			*			*		
Slope $\beta_1$	1.13	.95	.70	1.08	.67	.54	.52	.25	.32
Standard Error	.22	.26	.18	.19	.17	.12	.14	.19	.15
R-squared	.47	.34	.33	.54	.41	.39	.31	.14	.14
$\Delta$ R-squared	.04	.00	.03	.00	.04	.02	.00	.33	.02

\* Significance at the 5% level

all test lakes. Such a model has the form:

$$\log Y_t = \beta_0 + \beta_1 \log Y_r + \sum_{j=1}^g \alpha_j D_j + \sum_{j=1}^g \gamma_j D_j \log Y_r + \delta B + \sum_{j=1}^g f_j D_j B + e,$$

where each  $D_j$  is a dummy of indicator 0,1 variable depending upon whether or not the observation is from the  $j$ -th test lake or not, and  $B$  is a 0,1 variable for pre- or post-ban ( $g$  denotes the number of test lakes minus one). This model permits estimation of the differential effect on the slope and intercept for the various test lakes. In partitioning the test lake variability, the method of estimation removed the components due to the

indicator variables for the different test lakes and due to the reference lakes, before evaluating the ban component. Another way of saying this is that the log test lake response is being considered as the sum of a general intercept, a linear component relationship with the log reference lake response, an adjustment in the intercept for the specific test lake, an adjustment in the slope for the specific test lake, an adjustment in the general intercept for the pre/post ban and an adjustment in the intercept for a specific test lake for the pre/post ban with the analysis partitioning the test lake response variability into assignable sources in the order listed.

Table 4 provides a summary of the analysis of variance for each of total phosphorus (TP), Secchi disc depth (SD) and chlorophyll-a (CHLA). For the corrected total sum of squares, the variability was partitioned sequentially into the following components: reference lake, intercept adjustment for different test lakes, adjustment of slope of reference lake variables for different test lakes, and finally, intercept adjustment for post/pre-ban effect. Another way of expressing this is that one adjusts the total test lake response variability for potential relationship with the corresponding reference lake response and for individual test lake differences and then examines for the

TABLE 4.

Combined Covariate Analysis.

<u>Model Steps</u>	<u>Parameter</u>	<u>Sum of Squares</u>	<u>D.F.</u>	<u>Mean Square</u>	<u>Test Stat.</u>	<u>R<sup>2</sup></u>	<u>ΔR<sup>2</sup></u>
Reference lake Response	TP	3.17	1	3.17	61.0	.16	.16
	SD	1.71	1	1.71	71.0	.15	.15
	CHLA	2.43	1	2.43	15.9	.06	.06
Intercept Adjustment for Test Lakes	TP	5.10	6	.85	15.3	.41	.25
	SD	4.98	6	.83	50.3	.60	.45
	CHLA	8.18	6	1.36	8.9	.25	.19
Slope Adjustment for Test Lakes	TP	1.79	6	.30	5.8	.50	.09
	SD	.49	6	.082	5.0	.64	.04
	CHLA	1.08	6	.18	1.2	.28	.03
Intercept Adjustment for Post-/Pre- Ban	TP	.20	7	.03	.6	.51	.01
	SD	.72	7	.103	6.2	.71	.07
	CHLA	1.40	7	.20	1.3	.31	.03
Residual	TP	9.98	193	.052			
	SD	3.25	197	.0165			
	CHLA	29.11	190	.153			

effect of imposition of the ban. Only the Secchi disc depth measurement showed a detectable variation between the pre- and post-ban values at a five percent level of significance.

By inspection of the column under  $R^2$  in Table 4, one can assess the proportion of variability in the data explained by the model. The model appears to do much better in this respect for total phosphorus (.51) and Secchi disc depth (.71) than it does for chlorophyll-a (.31).

Some additional information of potential interest that can be obtained from Table 4 is the proportion of the variability explained by various groups of terms in the model. Those are summarized in Table 5.

TABLE 5.

Proportion of Variability Explained by Various Sources.

Source	Measurement	Proportion of Variability Explained by Model	Proportion of Total Variability
Reference	TP	0.31	0.16
Lake Covariate	SD	0.22	0.15
	CHLA	0.19	0.06
Test Lake	TP	0.63	0.34
Difference	SD	0.69	0.49
	CHLA	0.71	0.22

Table 6 lists each test lake's estimates of the slope coefficients for the corresponding reference lake as well as estimates of the amount of shift in the model after the imposition of the ban. The estimated standard deviations of these estimates are listed in parentheses. Asterisks (\*) are used to indicate statistical significance of at least the five percent level.

A shift associated with the ban was detectable at the five percent level in only 4 of the 21 cases, namely for total phosphorus in Enterprise Lake, Secchi disc depth in Elk and Townline Lakes, and chlorophyll-a in Elk Lake. In two of these cases (total phosphorus for Enterprise Lake and chlorophyll-a for Elk Lake) a positive direction in the post-ban shift is not something that could be attributable to the ban. Hence, from

TABLE 6. Estimates of the post-/pre-ban shift and slope-coefficient for corresponding reference lakes.

Lake -----	Total Phosphorus -----		Secchi Depth -----		Chlorophyll - a -----	
	Post/Pre chnng. in -----	Slope -----	Post/Pre chnng. in -----	Slope -----	Post/Pre chnng. in -----	Slope -----
Swan	.08 (.10)	-.13 (.23)	.17 (.17)	-.81 * (.28)	.31 (.17)	-.11 (.21)
Balsam	-.01 (.09)	.47 * (.17)	.13 (.16)	.51 * (.16)	-.19 (.16)	.33 (.21)
Butternut	-.01 (.09)	.37 * (.17)	.14 (.16)	.64 * (.16)	-.06 (.16)	.38 (.21)
Elk	-.04 (.09)	.35 * (.17)	.16 (.16)	.31 (.16)	.26 (.16)	.40 (.21)
Enterprise	.17 (.09)	1.14 * (.17)	0 (.16)	.75 * (.20)	.15 (.16)	.70 * (.21)
Moss	0 (.09)	1.11 * (.17)	.07 (.16)	.66 * (.20)	.07 (.16)	.54 * (.21)
Townline	-.02 (.04)	.54 * (.17)	.22 * (.06)	.24 (.21)	.10 (.16)	.32 (.21)

\* Statistically significant at at least the 5% level  
( ) Standard Deviation

this analysis, the only effect that appears to be associated with the ban is for Secchi disc depth. The relative magnitude of this shift is approximately 10 percent, and, although statistically significant, a question could be raised about the meaningfulness of its significance.

The number of slope estimates that are statistically significant is an indicator that the relationship of the reference lakes to the test lakes is accounting for a statistically significant proportion of the variability. These data were useful in making the analysis more sensitive. However, the amount of variability not explained by this relationship is larger still.

#### Multivariate Analysis/Multiple Comparisons

A general multivariate analysis taking into account the covariance structure of the data was carried out. It complements

the preceding two analyses by using statistical procedures which account for possible correlation of the measurements.

To this end, the measurements for a given lake and year were considered to be a single multivariate variable, or vector, for purposes of analysis. For each post-ban year, the vector analyzed actually consisted of the differences from the corresponding sampling times for the single pre-ban year. In one analysis, the test lakes and the reference lakes were considered together. In another analysis, the test lakes were considered separately. In either case, the vectors of differences were analyzed in a two-way table in which the entries were identified by lake and by year. Simultaneous confidence intervals on the differences were also constructed. The description of the procedures for these analyses are given in the Appendix.

A similar analysis was also performed for each test lake using the differences of the logarithms of the test lake measurements and the corresponding reference lake measurements, in a sense an analysis of the test lake data adjusted for a potential relationship with its corresponding reference lake.

The estimated differences for correspondings dates between post- and pre-ban measurements and their simultaneous confidence intervals are best presented graphically. Figure 3 shows the

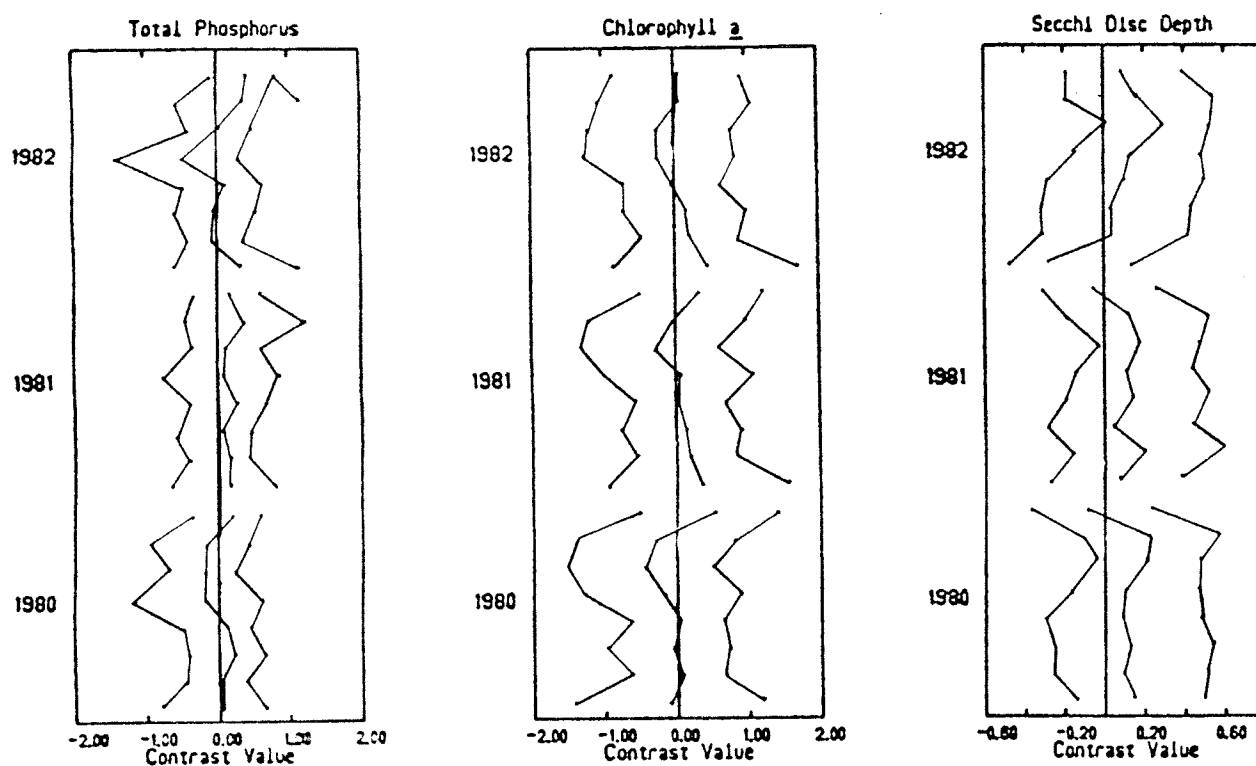


Figure 3. Estimated differences and 95% confidence bounds for post-ban effects. Analysis of data from all lakes.

results for the three analyses for total phosphorus, Secchi disk depth, and chlorophyll-a. Figure 4 presents a similar analysis for test lakes only. For each response variable and year, the left curve is the lower confidence bound, the middle curve the estimated contrast value, and the right curve the upper confidence bound. A vertical "no effect" line passes through zero.

It is clear from Figures 3 and 4 that the ban has not had a statistically significant effect on total phosphorus, chlorophyll-a, or Secchi disk depth, although the general positive nature of the estimate for the latter for all post-ban years may support an indication of some effect for Secchi disk depth.

This multivariate analysis was also performed for data constructed from the differences of the log test lake responses and the corresponding log reference lake responses. Graphs of the simultaneous confidence intervals on the differences between post- and pre-ban years for each point in time that was sampled are given in Figure 5. In this analysis no effect of the ban is observable.

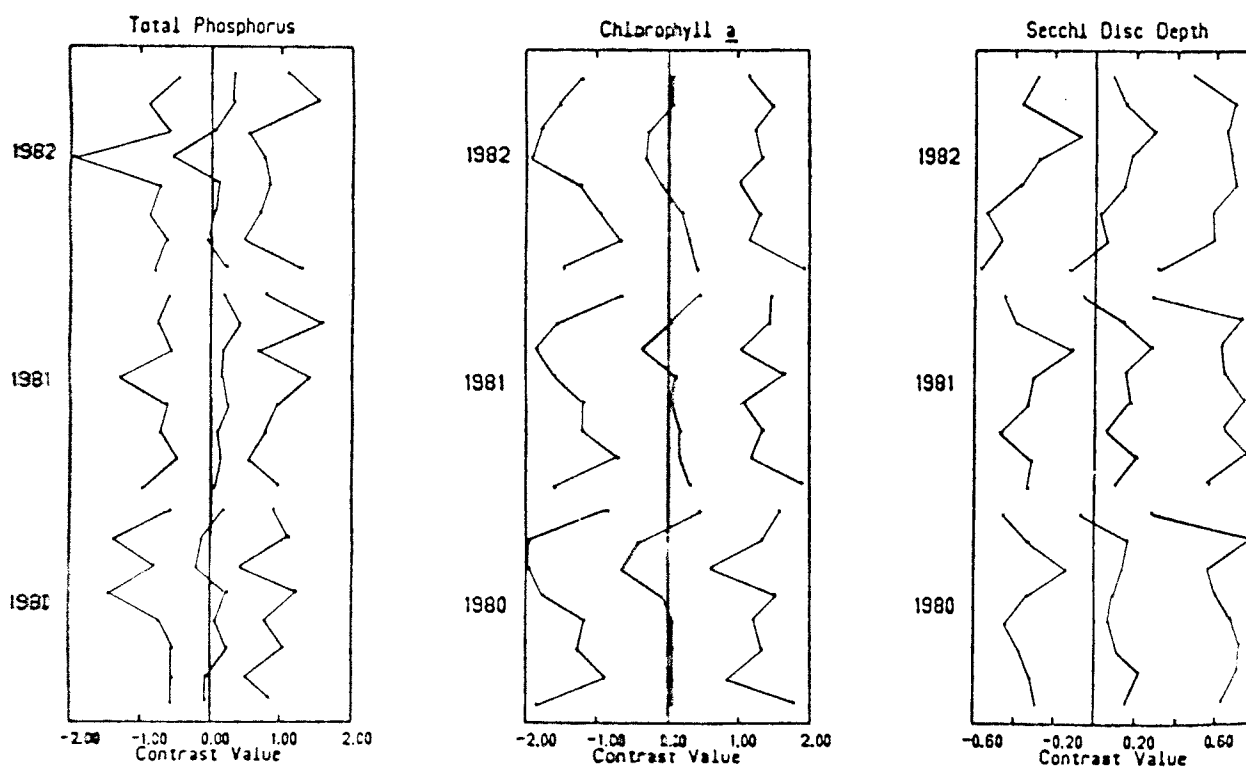


Figure 4. Estimated differences and 95% confidence bounds for post-ban effects. Analysis of test lakes only, logarithmic transform of data.

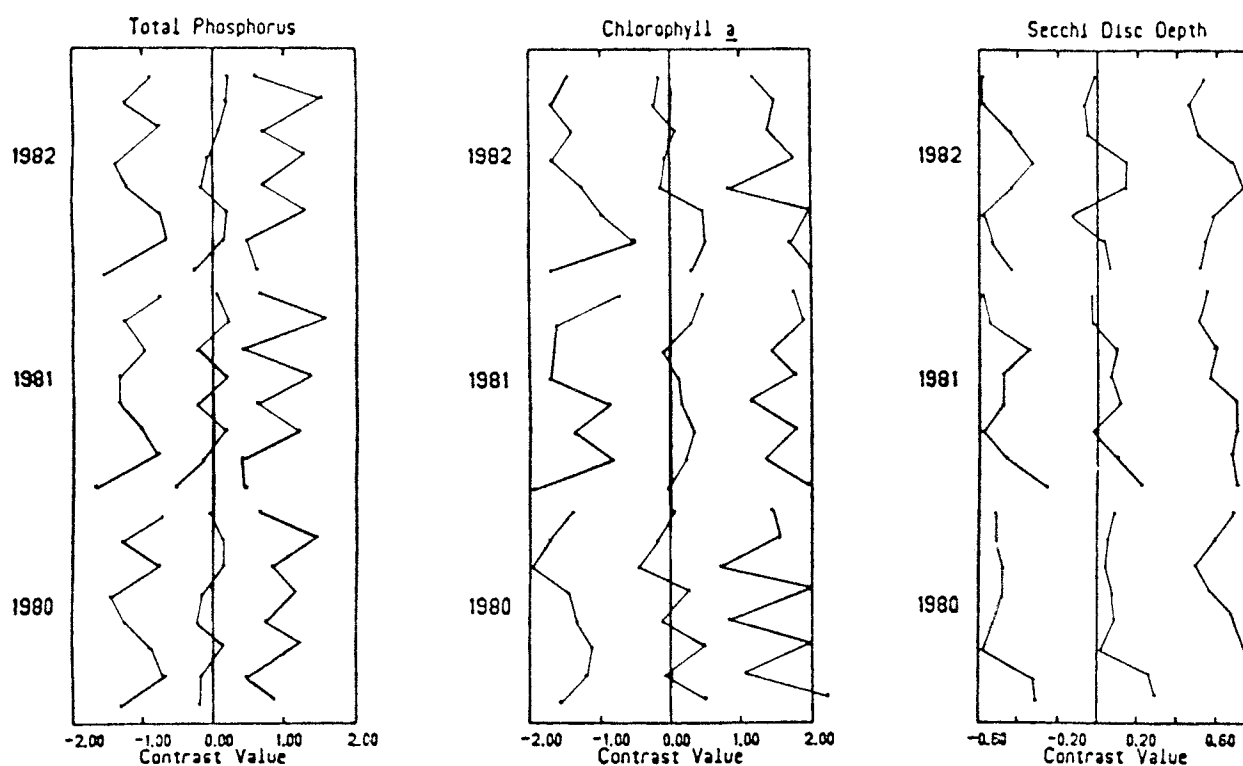


Figure 5. Estimated differences and 95% confidence bounds for post-ban effects. Analysis of difference of logarithmic transform of data between test lakes and reference lakes.

### CONCLUSION

An effect of the phosphate ban, if any, was sufficiently small that its detection with statistical significance was not possible with the amount of variability observed in the data. However, it would appear that models involving reference lake measurements had their sensitivity improved for detecting ban effects. One could use this improvement to estimate the amount of additional test lake measuring that would be needed to provide the same sensitivity if one chose to eliminate sampling the reference lakes.

The multivariate/multiple comparison analysis, based upon assumptions that are more supportable, would only have been capable of detecting a ban effect if there had been much more data or the measurement variability was greatly reduced. It does permit a valid analysis without the necessity of using a model relating the response to the time of year. Of course, with sufficient frequency of sampling over time to permit reliable estimation of such a model, considerably greater power for detection of a ban effect would result.

## APPENDIX

## Description of the Multivariate/Multiple Comparison Analysis

Logarithms of the measurements for each response variable for a lake in a year were analyzed as an eight-dimensional vector response. These data were then analyzed as a multivariate two-way layout.

The model can be written mathematically as:

$$X_{ijk} = \mu_k + Y_{ik} + L_{jk} + e_{ijk}$$

( $i = 1, 2, 3, 4$ ,  $j = 1, 2, 3, 4, 5, 6, 7, 8, 9$ ,  $k = 1, 2, 3, 4, 5, 6, 7, 8$ )

where  $X_{ijk}$  is the  $k^{\text{th}}$  observation on lake  $j$  in year  $i$ ,  $\mu_k$  is the  $k^{\text{th}}$  component of the grand mean,  $Y_{ik}$  is the  $k^{\text{th}}$  component of the effect of year  $i$ , and  $L_{jk}$  is the  $k^{\text{th}}$  component of the effect of lake  $j$ . The errors  $\{e_{ij1}, e_{ij2}, e_{ij3}, e_{ij4}, e_{ij5}, e_{ij6}, e_{ij7}, e_{ij8}\}$  are assumed to be independent eight-variable Gaussian with zero mean and covariance matrix  $\Sigma$ . There are two principle advantages of this model:

1. It takes account of the covariance structure of the data.
2. It is simple, allowing for differences between lakes and years without assuming a specific mathematical model for the difference.

## Parameter Estimation

The multivariate analysis of variance closely parallels its univariate counterpart. Maximum likelihood estimates of the effects are given by:

$$\begin{aligned}\hat{\mu}_k &= \bar{X}_{..k} \\ \hat{Y}_{ik} &= \bar{X}_{i.k} - \bar{X}_{..k} \\ \hat{L}_{jk} &= \bar{X}_{.jk} - \bar{X}_{..k},\end{aligned}$$

where the dot and bar denote averaging over subscripts. The maximum likelihood estimate,  $\hat{\Sigma}$ , of the error covariance matrix, is proportional to the error sum of squares and cross products matrix  $E$ , where

$$E_{kg} = \sum_{i=1}^4 \sum_{j=1}^9 (X_{ijk} - \hat{\mu}_k - \hat{Y}_{ik} - \hat{L}_{jk})(X_{ijg} - \hat{\mu}_g - \hat{Y}_{ig} - \hat{L}_{jg}).$$

The statistical tests of interest are multiple comparisons of contrasts  $C_{ik} = Y_{ik} - Y_{1k}$ , denoting the difference in measurement  $k$  between post-ban year  $i$  (1980, 1981, or 1982) and the pre ban year, 1978. The  $C_{ik}$  are estimated by  $\hat{C}_{ik} = \hat{Y}_{ik} - \hat{Y}_{1k}$ ,  $i = 1, 2, 3, 4$ ,  $k = 1, 2, 3, 4, 5, 6, 7, 8$ .



By formula (8), pp. 200-201 of Morrison (1976), the 100 (1-  $\alpha$ ) percent simultaneous confidence intervals on the  $\{C_{ik}\}$  for all nine lakes are:

$$C_{ik} - \left( \frac{2E_{kk}}{9} \cdot \frac{X_{\alpha}}{1-X_{\alpha}} \right)^{\frac{1}{2}} \leq C_{ik} \leq \hat{C}_{ik} + \left( \frac{2E_{kk}}{9} \cdot \frac{X_{\alpha}}{1-X_{\alpha}} \right)^{\frac{1}{2}}$$

Here  $X_{\alpha}$  is the upper 100 $\alpha$  percentage point of the greatest characteristic root distribution with parameters (in Morrison's notation),  $s = 3$ ,  $m = 2$ , and  $n = 7.5$ . We take  $\alpha = 0.05$ , and find from Chart 11, p. 381 of Morrison that  $X_{\alpha} = 0.665$ . Although the above is for all nine lakes, similar expressions can be displayed for the other situations discussed in the Methods Section.

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